Nonlinear model based simulation of the economical growth of two unisectorial interacting economies

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Abstract: This paper is focused on the simulation of the economical growth of two unisectorial interacting economies in a framework of a Kaldor model. Simulation reveals significant differences between the case of non balanced and balanced growth.

Keywords: economical growth, unisectorial, interacting economies

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1. THE MODEL

In North-South model presented in [1] there are two regions and two goods: one manufactured in North and the other agricultural produced in South.

The economy in North is characterized by:

- uses: labour force and capital;
- product: a manufactured good:
  - sold in South,

- consumed in North,
- reinvested in North.

In South the economy is characterized by:

- uses: labour force, capital, and land,
- product: an agricultural good:
  - sold in North,
  - consumed in South,
  - reinvested in South.

Assumptions for both economies:

- the level of output is determined by supply;
- real wages rate are fixed exogenously in both economies (w_s, w_n);
- labour force is unlimited;
- all wages are consumed;
- all profit is reinvested.

Special assumptions for South economy:

- all rents are consumed;
- there is enough land;
- the rent (r) is fixed exogenously.

The North economy

The production function is assumed to be of Cobb-Douglas type:

$$Q_n\left(L_n(t), K_n(t)\right) = \eta \cdot L_n^{\theta}(t) \cdot K_n^{1-\theta}(t) \tag{1}$$

where:

- n designs North;
- $\eta>0$, $0<\theta<1$ positive constant parameters;
- $L_n(t)\geq 0$, $K_n(t)\geq 0$;
- Q_n = output [lei]
- lei=Romanian domestic currency unit
- L_n = labour force $\left[\text{man - years}/\text{year}\right]$

➲ K_n = capital $\left[{}^{\text{machine - years}}\!\!\diagup_{\text{year}}\right]$

➲ η= parameter $\left[\dfrac{\text{lei}}{\left({}^{\text{man - years}}\!\!\diagup_{\text{year}}\right)^{1-\theta}\left({}^{\text{machine - years}}\!\!\diagup_{\text{year}}\right)^{\theta}}\right]$.

Assumption 1: The consumption in North economy is composed from manufactured good produced in North, and agricultural good produced and bought from South.

The equation which describes the growth of the capital in North is:

$$\dot{K}_n(t) = \sigma_n \cdot \left(1 - \theta \cdot p^{1-\delta}(t)\right) K_n(t) \tag{2}$$

where:

1. $$\sigma_n = \left(\eta \cdot \left(\frac{\theta}{w_n}\right)^{\theta}\right)^{\frac{1}{1-\theta}} \tag{3}$$
2. δ is the proportion of wages spent on the manufactured good, $0<\delta<1$;
3. $p = \dfrac{P_s}{P_n}$ is the term of trade which connects the two economies, Ps is the price for agricultural good from South and P_n is the price for the manufactured good produced in North, $0<p<1$;
4. the preferences of the consumer are of Cobb-Douglas type, and therefore the cost of living index is given by: $\Phi = P_n^{\delta} \cdot P_s^{1-\delta}$ (4)

The output Q_n, labour force L_n, consumption C_n, profit Π_n are given by:

5. $$Q_n = \sigma_n \cdot K_n \tag{5}$$
6. $$L_n = \frac{\theta}{w_n} \cdot \sigma_n \cdot K_n \tag{6}$$
7. $C_n = \sigma_n \cdot \theta \cdot (1-\delta) \cdot p^{1-\delta} \cdot K_n$ the consumption for agricultural good [lei] (7)
8. $\Pi_n = I_n = \sigma_n \cdot \left(1 - \theta \cdot p^{1-\delta}\right) K_n$ the profit and investment in North (8)

The South economy

The production function is assumed to be of Coob-Dougls type:

$$Q_s\left(L_s(t), K_s(t), Z_s(t)\right) = \Theta L_s^{(1-\alpha-\beta)}(t) K_s^{\alpha}(t) Z_s^{\beta}(t) \tag{9}$$

where:

- s designs South;
- Θ, α, $\beta > 0$ positive constant parameters;
- $0<\alpha+\beta<1$;
- $L_s(t)\geq 0$, $K_s(t)\geq 0$, $Z_s(t)\geq 0$.
- Q_s = output [lei]
- Ls = labour force $\left[{}^{\text{man}-\text{years}}\!\big/_{\text{year}}\right]$
- Ks = capital $\left[{}^{\text{machine}-\text{years}}\!\big/_{\text{year}}\right]$
- Zs = land $\left[{}^{\text{acres}-\text{years}}\!\big/_{\text{year}}\right]$
- Θ = parameter

$$\left[\frac{\text{lei}}{\left({}^{\text{man}-\text{years}}\!\big/_{\text{year}}\right)^{1-\alpha-\beta}\left({}^{\text{machine}-\text{years}}\!\big/_{\text{year}}\right)^{\alpha}\left({}^{\text{acres}-\text{years}}\!\big/_{\text{year}}\right)^{\beta}}\right]$$

Assumption 2: Wages and rents are consumed entirely and consumption in South is restricted to the agricultural good.

Assumption 3: Profit is assumed to be entirely used for purchasing the manufactured good produced in North, which is invested and used in the production process:

$$I_s = p \cdot \Pi_s \tag{10}$$

where p is the term of trade, and I_s investments in South.

The basic equation which describes the growth mechanism in South is:

$$\dot{K}_s(t) = \alpha \cdot \sigma_s \cdot p(t) \cdot K_s(t) \tag{11}$$

where:

1. $\sigma_s = \left(\Theta \cdot \lambda^{1-\alpha-\beta} \cdot \left(\frac{\beta}{r}\right)^{\beta}\right)^{\frac{1}{\alpha}} = \text{constant}$ (12)

2. $\lambda = \frac{1-\alpha-\beta}{w_s}$ (13)

The output Q_s, labour force L_s, profit Π_s, investment I_s, consumption C_s, land Z_s are given by:

3. $Q_s = \sigma_s \cdot K_s$ (14)
4. $L_s = \lambda \cdot \sigma_s \cdot K_s$ (15)
5. $Z_s = \frac{\beta}{r} \cdot \sigma_s \cdot K_s$ (16)
6. $\Pi_s = \alpha \cdot \sigma_s \cdot K_s$ (17)
7. $I_s = p \cdot \alpha \cdot \sigma_s \cdot K_s$ (18)

Market clearing and the terms of trade

Assumption 4: In North and South the total expenditure are equal with the total income and there is no accumulation of financial goods (money, bonds).

Because the model contains only two economies with two goods (manufactured and agricultural), therefore by Walras'law, it is enough to write the market clearing condition just for one of the markets. For example, the market clearing condition implies that the surplus over Southern consumption is purchased by the North.

$$C_n = \Pi_s \tag{19}$$

Using (19), (17) and (7) the equation for the trade is:

$$p(t) = \left[\frac{(1-\delta)\cdot \theta \cdot \sigma_n}{\alpha \cdot \sigma_s}\right]^{1/\delta} \cdot \left(\frac{K_n(t)}{K_s(t)}\right)^{1/\delta} \tag{20}$$

The system of differential equations which governs the evolution of capitals

With (1), (9) and (20) we determine the system of differential equations which defines the model:

$$
\begin{cases}
\dot{K}_s(t, p(t)) = \alpha \cdot \sigma_s \cdot p(t) \cdot K_s(t) \\
\dot{K}_n(t, p(t)) = \sigma_n \cdot \left(1 - \theta \cdot p(t)^{1-\delta}\right) K_n(t) \\
p(t) = \left[\frac{(1-\delta) \cdot \theta \cdot \sigma_n}{\alpha \cdot \sigma_s} \right]^{1/\delta} \cdot \left(\frac{K_n(t)}{K_s(t)} \right)^{1/\delta}
\end{cases}
\tag{21}
$$

where K_n, K_s and p are unknown functions, and _, _n, _s, _ and _ are constants assumed to be known.

Substituting p(t) we obtain:

$$
\begin{cases}
\dot{K}_n(t) = \sigma_n \cdot K_n(t) - \sigma_n^{1/\delta} \cdot \theta^{1/\delta} \cdot (1-\delta)^{\frac{1-\delta}{\delta}} \cdot \alpha^{\frac{\delta-1}{\delta}} \cdot \sigma_s^{\frac{\delta-1}{\delta}} \cdot K_n^{1/\delta}(t) \cdot K_s^{\frac{\delta-1}{\delta}}(t) \\
\dot{K}_s(t) = \alpha^{\frac{\delta-1}{\delta}} \cdot \sigma_s^{\frac{\delta-1}{\delta}} \cdot (1-\delta)^{1/\delta} \cdot \theta^{1/\delta} \sigma_n^{1/\delta} \cdot K_n^{1/\delta}(t) \cdot K_s^{\frac{\delta-1}{\delta}}(t)
\end{cases}
\tag{22}
$$

This system determines the evolution of the capitals ($K_n(t)$, $K_s(t)$) in both economies, from which we can find the other elements of the growth ($Q_n(t)$, $Q_s(t)$, $L_n(t)$, $L_s(t)$, $\Pi_n(t)$, $\Pi_s(t)$, $I_n(t)$, $I_s(t)$, p(t) and $Z_s(t)$).

2. BALANCED GROWTH

We will investigate the behavior of the system setting the condition for balanced growth equilibrium, where both economies grow at the same rate.

From (2) and (11):

$$
\begin{cases}
\dot{K}_s(t) = \alpha \cdot \sigma_s \cdot p(t) \cdot K_s(t) \\
\dot{K}_n(t) = \sigma_n \cdot \left(1 - \theta \cdot p^{1-\delta}(t)\right) K_s(t)
\end{cases}
\tag{23}
$$

follows:

$$
\begin{cases}
\hat{K}_s = \alpha \cdot \sigma_s \cdot p \\
\hat{K}_n = \sigma_n \cdot \left(1 - \theta \cdot p^{1-\delta}\right)
\end{cases}
\tag{24}
$$

where ^ designs the growth rate of a variable:

$$\hat{K}_s = \frac{\dot{K}_s}{K_s} \text{ _i } \hat{K}_n = \frac{\dot{K}_n}{K_n} \tag{25}$$

Let $$k = \frac{K_n}{K_s} \tag{26}$$

We define relative capital growth $\hat{k}$:

$$\hat{k} = \hat{K}_n - \hat{K}_s \tag{27}$$

therefore

$$\hat{k} = \sigma_n - \theta \cdot \sigma_n \cdot p^{1-\delta} - \alpha \cdot \sigma_s \cdot p \tag{28}$$

Equations (20) and (28) can be used to obtain the equilibrium values k^* and p^* for $\hat{k}$=0.

$$\begin{cases} p - \left[\dfrac{(1-\delta)\cdot\theta\cdot\sigma_n}{\alpha\cdot\sigma_s}\right]^{1/\delta} \cdot (k)^{1/\delta} = 0 \\ \sigma_n - \theta\cdot\sigma_n\cdot p^{1-\delta} - \alpha\cdot\sigma_s\cdot p = 0 \end{cases} \tag{29}$$

where k and p are unknown.

3. NUMERICAL SIMULATION

CASE 1

We consider the case defined by the following values of the parameters Θ, α, β, w_s, r, w_n, η, θ, δ:

- Θ=9
- β=0.3
- α=0.4
- w_s=10
- r=0.02
- w_n=15
- η=6
- θ=0.3

➲ $\delta=0.3$

The numerical code used for simulation was made in Mathematica 4.0.

For these values we have:

➲ $\sigma_s=133.511$

➲ $a=0.02$

➲ $\sigma_n=2.41832$

➲ $\lambda=0.03$

For the initial values of capitals:

➲ $K_{s0}=5*10^{12}$

➲ $K_{n0}=10*10^{12}$

we find the evolutions plotted in the Fig. 1-6:

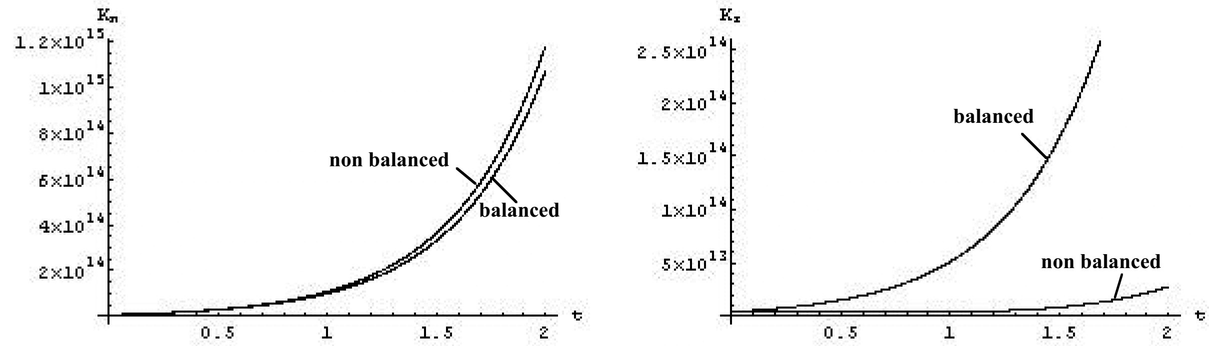

Fig.1. The evolution of capitals $K_n(t)$ and $K_s(t)$

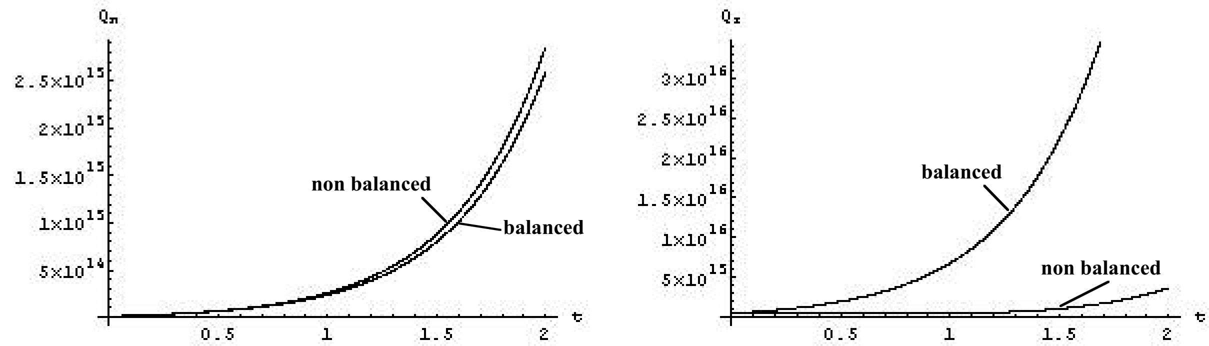

Fig.2. The evolution of the productions $Q_n(t)$ and $Q_s(t)$

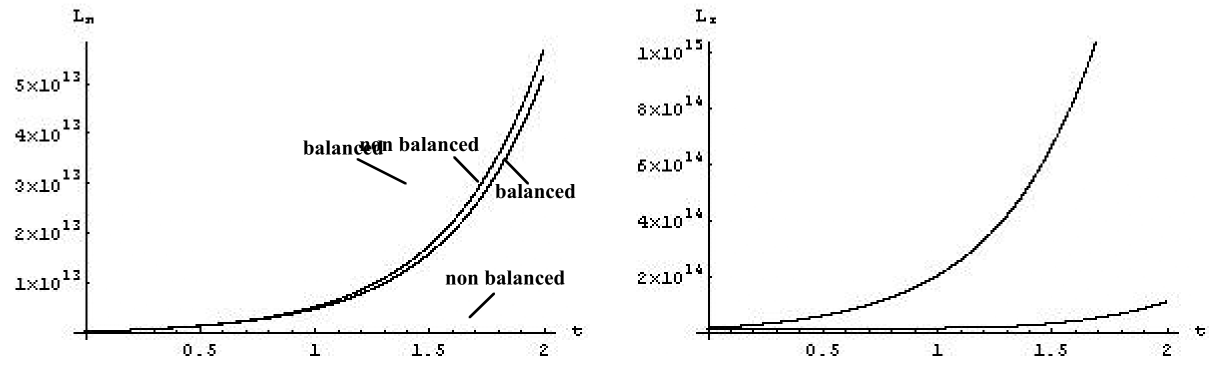

Fig.3. The evolution of the labour forces $L_n(t)$ and $L_s(t)$

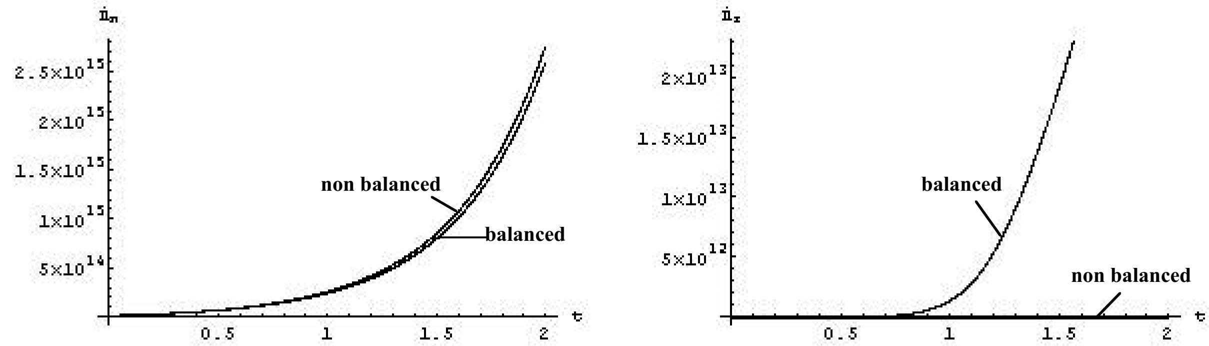

Fig.4. The evolution of the investments $I_n(t)$ and $I_s(t)$

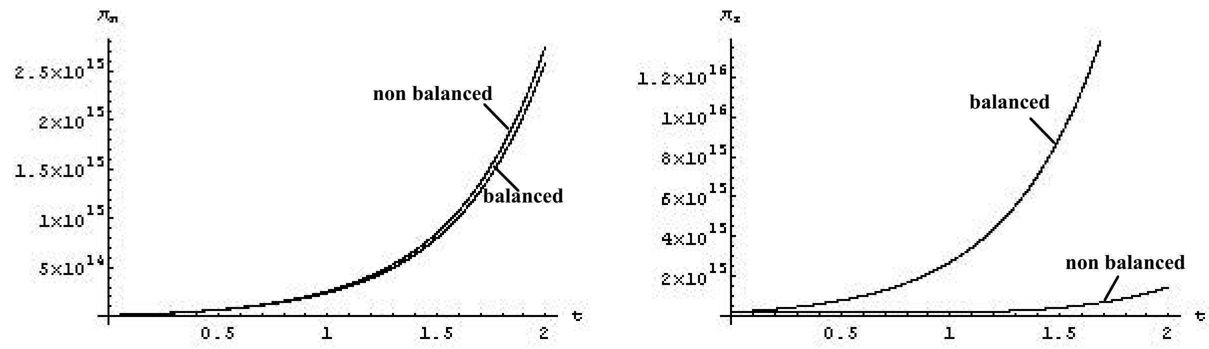

Fig.5. The evolution of the profits $\Pi_n(t)$ and $\Pi_s(t)$

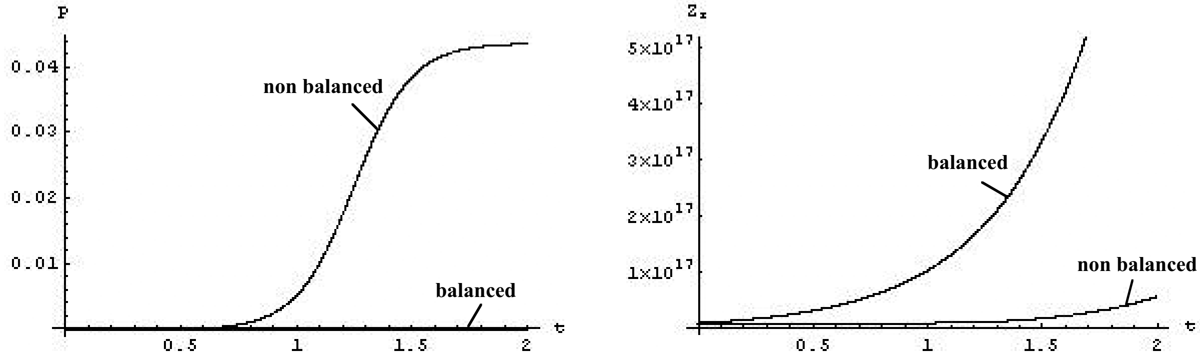

Fig. 6. The evolution of the term of trade p(t) and land $Z_s(t)$

CASE 2

We consider the case defined by the ***same values of the parameters*** Θ, α, β, ws, r, wn, η, θ, δ as in Case 1 and for the initial values of the capital:

- K_{s0}=8.5340243*10^6
- K_{n0}=1,9384474*10^6

we find the following evolutions plotted in Fig. 7-12:

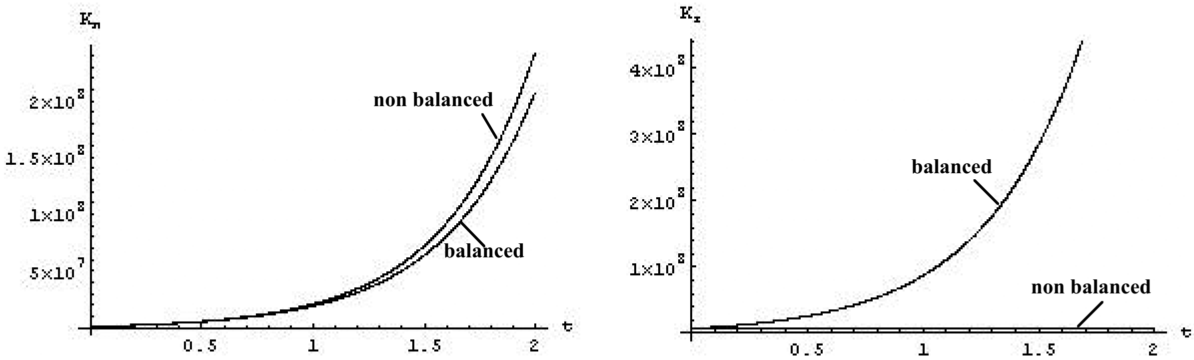

Fig.7. The evolution of capitals $K_n(t)$ and $K_s(t)$

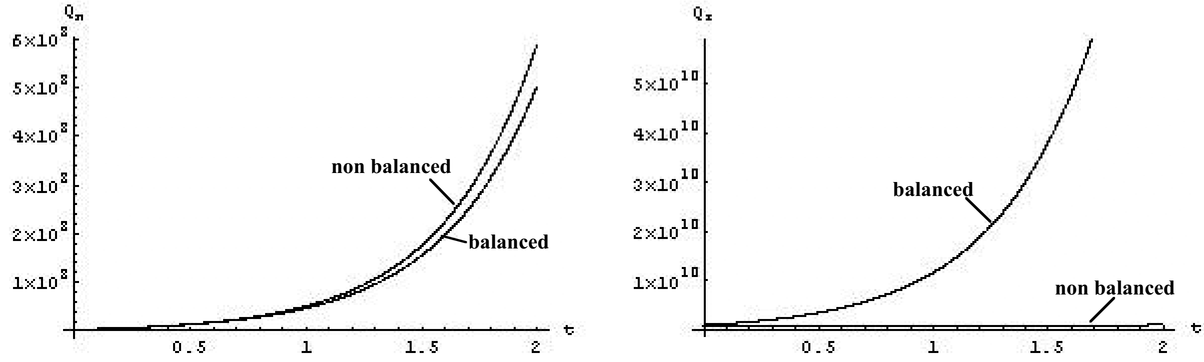

Fig.8. The evolution of the productions $Q_n(t)$ and $Q_s(t)$

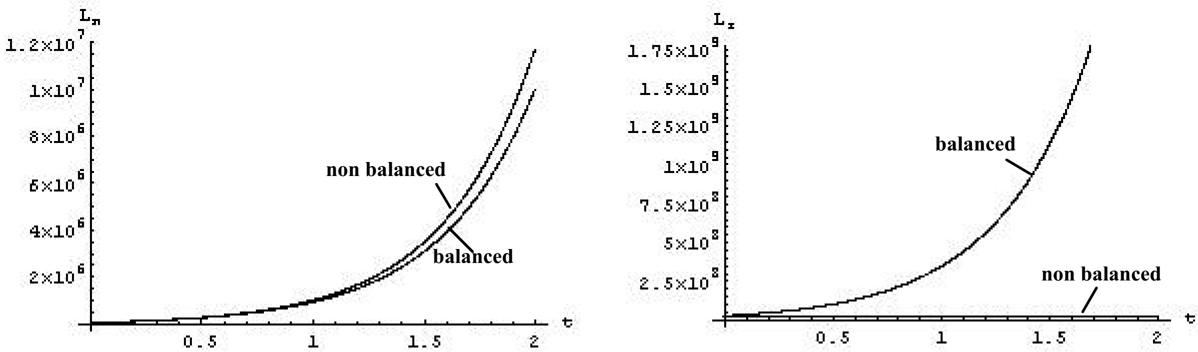

Fig.9. The evolution of the labour forces $L_n(t)$ and $L_s(t)$

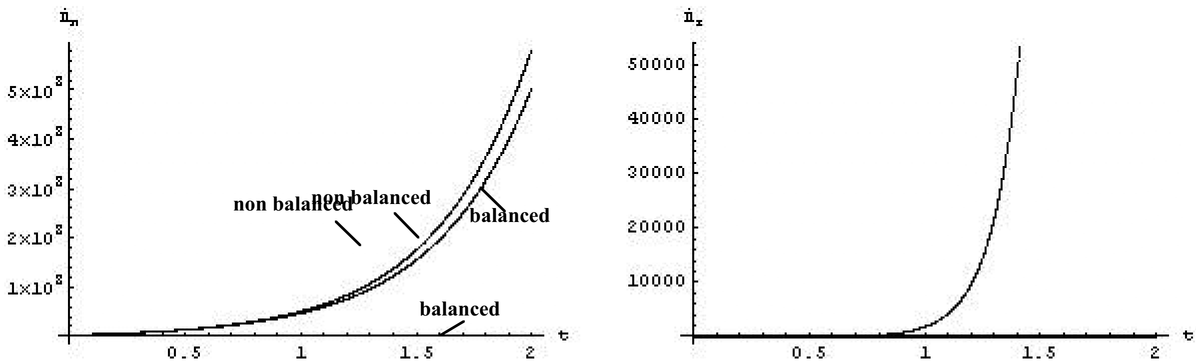

Fig.10. The evolution of the investments $I_n(t)$ and $I_s(t)$

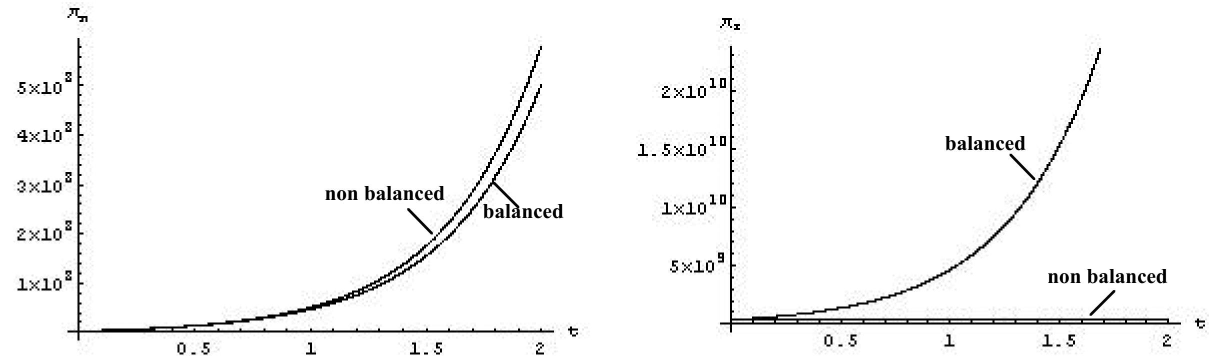

Fig.11. The evolution of the profits $\Pi_n(t)$ and $\Pi_s(t)$

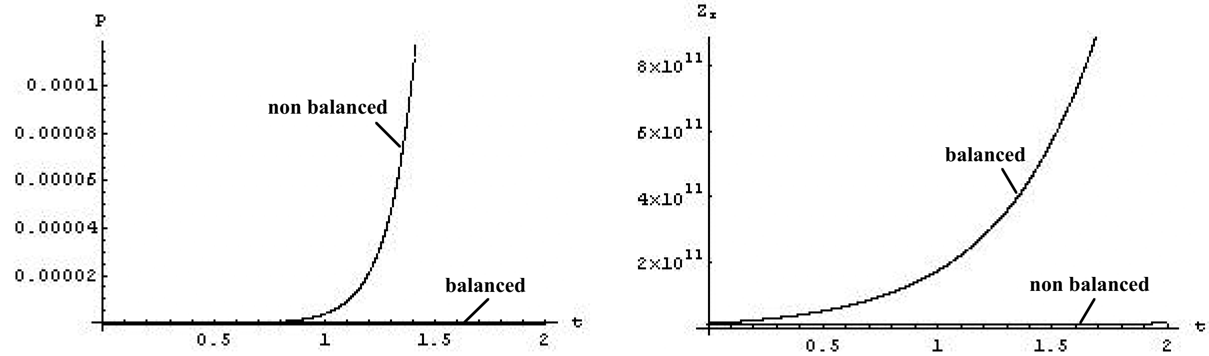

Fig. 12. The evolution of the term of trade p(t) and land $Z_s(t)$

4. CONCLUSIONS

1. Capitals K_n, K_s; productions Q_n, Q_s; labour forces L_n, L_s investment I_n, I_s and profit Π_n, Π_s, land Z_s and the term of trade p increase in both economies;
2. The increase of K_s, Q_s, L_s, I_s, Π_s, Z_s in the first time unit is very small. As smaller the initial values $K_n(0)$, $K_s(0)$ are, as longer is the period in which the variations of K_s, Q_s, L_s, I_s, Π_s, Z_s are small.
3. Simulation reveals significant differences between the computed values of K_s, Q_s, L_s, I_s, Π_s, Z_s in the case of non balanced and balanced growth.

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