On the dynamical behavior of the exchange rate in a nonlinear model

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Abstract. The main objective of this paper is to put in evidence the chaotic behavior of the dynamic of exchange rate in a nonlinear model.

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1. Introduction

In this paper we are interested in the study of the following dynamical system (see [2], [3]):

$$S_t = S_{t-1}^{[(c+\alpha)bm_t+(1-\alpha)b]} S_{t-2}^{-cm_t}, t \in Z \tag{1}$$

which describes the exchange rate evolution. S_t is the exchange rate at the moment *t*; *b* is the discount factor that speculators use to discount the future expected exchange rate (0<b<1). The parameter b measures the influence of expected future exchange rate value on its present value. m_t is the weight given by chartists and it is defined by:

(2) $$m_t = \frac{1}{1+\beta\left(S_{t-1} - S_{t-1}^*\right)}, \beta > 0$$

The parameter β determines the speed with which the weight of the chartists declines and measures the degree of divergence of the fundamentalists' estimates from the equilibrium exchange rate. It is the parameter that measures the precision degree of the fundamentalists' estimates (see [2], [3]). When the exchange rate is in the neighborhood of the equilibrium rate, chartists' behavior dominates. If the fundamentalists observe a deviation today, then they expect that the market exchange rate to return to the fundamental equilibrium exchange rate with the speed $\alpha > 0$ during the next period.

The fundamentalists expect the market rate to return to that fundamental rate S_t^* with the speed α during the next period, if they observe a deviation today

We consider the case $c > 1$. In the particular case in which $c = 2$, the power spectrum and the fractal dimension of the attractor of this model have been partially studied (in [3]).

Mathematical and numerical results (for the same situation $c = 2$) are presented in other works (see [5], [6], [9]). There are proof when the fixed point of the system (1) is stable or unstable. In the case in which the fixed point is stable, for certain values of parameters it is shown that the fix point is locally or globally attractive. The existence of periodical cycle of period 2 is also investigated. Numerical simulation reveals situations in which the evolution is chaotic, periodical, quasi-periodical. Situations in which chaotic and non chaotic attractors coexist are find.

This paper is focused to find how the dynamics of the system (1) is changed when the parameter c is changed.

2. Some qualitative properties of the dynamical system defined by (1)

If we denote $s_t = \ln S_t$, then the system (1) can be rewritten in the following way:

(3) $$s_t = \left(\frac{(c+\alpha)b}{1+\beta\left(e^{s_{t-1}}-1\right)^2} + (1-\alpha)b \right) s_{t-1} - \frac{cb}{1+\beta\left(e^{s_{t-1}}-1\right)^2} s_{t-2}$$

with $s_t \in R, \forall t \in Z$.

If we introduce the function $F : R^2 \to R^2$, $F(x,y) = (F_1(x,y), F_2(x,y))$ with $F_1(x,y) = y$ and $F_2(x,y) = \varphi(y)y + \phi(y)x$ in which

$$\varphi(y) = \left(\frac{(c+\alpha)b}{1+\beta\left(e^{y}-1\right)^2} + (1-\alpha)b \right) \text{ and } \phi(y) = -\frac{cb}{1+\beta\left(e^{y}-1\right)^2}.$$

we can present (3) in the form:

(4) $$(s_t, s_{t+1}) = F(s_{t-1}, s_t)$$

The forms (3) and (4) are used at different steps in the study of the system (1). The form (4) is necessary for obtain the eigenvalues of the Jacobean matrix of the function F, to obtain also the Lyapunov exponents of the system and in study of periodical points of the system.

Proposition 1 The system (4) has a unique fix point (0,0). This point (0, 0) is stable for $b \in \left(0, \frac{1}{c}\right)$ and unstable if $b \in \left(\frac{1}{c}, 1\right)$.

Proposition 2 For $\alpha \in \left(0,\ 1+\frac{1}{b}\right]$ or for $\alpha \in \left(1+\frac{1}{b},\infty\right)$ and $\beta \in \left(0, \frac{(c+1)(\alpha-1)b^2+(1+cb)}{\left[(\alpha-1)^2 b^2-1\right]}\right]$

the system (4) has not periodical points of period 2.

Proposition 3 For b,α,β where $b \in (0,1), \alpha \in \left(1+\frac{1}{b},\infty\right)$ and

$\beta \in \left(\frac{(c+1)(\alpha-1)b^2+(1+cb)}{\left[(\alpha-1)^2 b^2-1\right]},\infty\right)$ the system (4) has only one periodical cycle of period 2. A point (s_0,s_1) belong to this cycle if and only if s_0 and s_1 are solution for the equation:

$$1=\frac{(c+1)b+(1-\alpha)b\beta\left(e^{\frac{(c+1)b+(1-\alpha)b\beta\left(e^x-1\right)^2}{1+cb+\beta\left(e^x-1\right)^2}x}-1\right)^2}{1+cb+\beta\left(e^{\frac{(c+1)b+(1-\alpha)b\beta\left(e^x-1\right)^2}{1+cb+\beta\left(e^x-1\right)^2}x}-1\right)^2}\,\frac{(c+1)b+(1-\alpha)b\beta\left(e^x-1\right)^2}{1+cb+\beta\left(e^x-1\right)^2}$$

Proposition 4. In the conditions of Proposition 3, if (s_0,s_1) belong to the periodical cycle, then:

i) if $\beta \in \left(\frac{(c+1)(\alpha-1)b^2+(1+cb)}{\left[(\alpha-1)^2 b^2-1\right]}, \frac{1+(1+2c)b}{\left[(\alpha-1)b-1\right]}\right)$ then

$s_0 > \ln\left(1+\sqrt{\frac{1}{\beta}\frac{1+(1+2c)b}{\left[(\alpha-1b-1)\right]}}\right)$ and $s_1 < -\ln\left(1+\sqrt{\frac{1}{\beta}\frac{1+(1+2c)b}{\left[(\alpha-1b-1)\right]}}\right)$

ii) if $\beta \in \left(\frac{1+(1+2c)b}{\left[(\alpha-1)b-1\right]},\infty\right)$ then

$$s_0 \in \left(\ln\left(1+\sqrt{\frac{1}{\beta}\frac{1+(1+2c)b}{[(\alpha-1b-1)]}}\right), -\ln\left(1-\sqrt{\frac{1}{\beta}\frac{1+(1+2c)b}{[(\alpha-1b-1)]}}\right)\right) \text{ and}$$

$$s_1 \in \left(\ln\left(1-\sqrt{\frac{1}{\beta}\frac{1+(1+2c)b}{[(\alpha-1b-1)]}}\right), -\ln\left(1+\sqrt{\frac{1}{\beta}\frac{1+(1+2c)b}{[(\alpha-1b-1)]}}\right)\right)$$

3. Numerical simulations.

We consider now a particular case when there are no periodical points of period 2. We take $\alpha = 2, b = 0.95, \beta = 6.2$, the initial condition is $(s_0, s_1) = (0.02, -0.02)$ and c is considered variable. We are interested to know the evolution of the system, in this particular case.

If $c \in (1,\ 1.05)$, then the trajectory tends to the fix point (0,0). For $c \in (1.05,\ 1.25)$, the trajectory tends to a limit cycle (quasi-periodical behavior).

For $c \in (1.25,\ 10)$ the system (1) display a chaotic behavior. Some of the computed results are plotted in (s_t, s_{t+1}) space on the Figures 1-6, the computation was made using the software *Mathematica*. A measure of the average rate of exponential divergence exhibited by a chaotic system is given by the Lyapunov exponents of the system; the positivity of such exponents can suggest the presence of chaos. The values of the Lyapunov exponents λ_1 and λ_2, given for the strange attractors represented in the Figures 1-6, confirm that the system (1) has in these cases a chaotic evolution. In order to obtain the values of the Lyapunov exponents we have used the method proposed in [4] and Householder QR factorization (see [7]). To estimate the values of these exponents, we have used an implementation with *Macros-Visual Basic for Application* (see [1]).

For $c > 10$, we observe that $|s_t| \to \infty$, when $t \to \infty$.

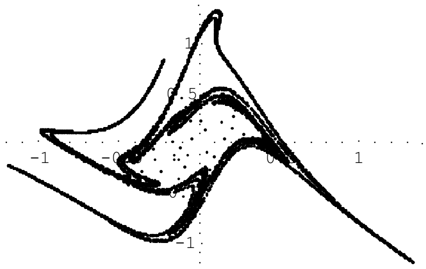

Fig.1: $c = 1.3, \lambda_1 = 0.2284, \lambda_2 = -0.7987$

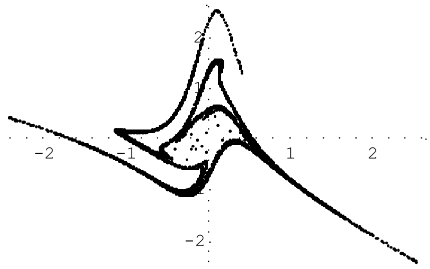

Fig. 2: $c = 1.45, \lambda_1 = 0.2575, \lambda_2 = -1.2057$

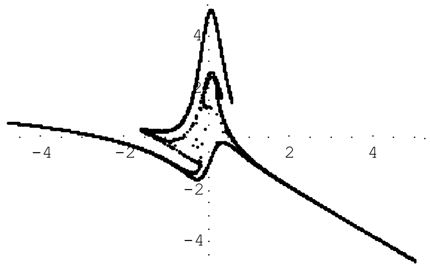

Fig. 3: $c = 2, \lambda_1 = 0.2646, \lambda_2 = -1.9035$

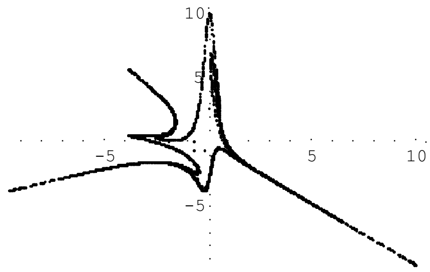

Fig.4: $c = 4, \lambda_1 = 0.1978, \lambda_2 = -1.701$

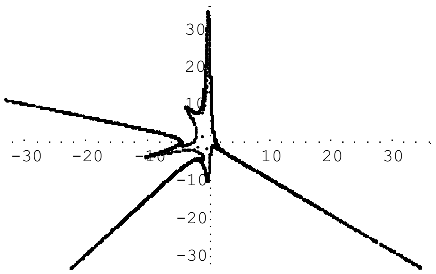

Fig.5: $c = 8, \lambda_1 = 0.1816, \lambda_2 = -4.4715$

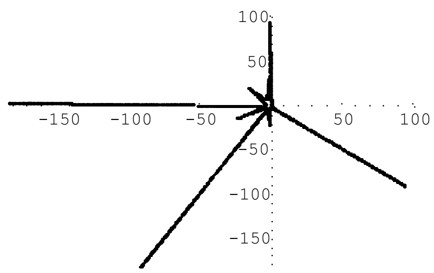

Fig.6: $c = 10, \lambda_1 = 0.1304, \lambda_2 = -15.624$

Conclusion

1. The main conclusion is that c is a major parameter, which influence the dynamic of the exchange rate in this model.

2. The model of chartists (see [2]) say that if the chartists observe today an exchange rate depreciation (appreciation) then they expect for the following period an other depreciation (appreciation) and that when c it is height these depreciations (appreciations) are strong. We can remark this influence of parameter c in the examples present in Section 3.

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