Using a new distribution of probability to model the number of claims in an automobile insurance portfolio

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Resumen

Negative Binomial distribution has been traditionally used in social sciences in order to model the number of claims. This distribution can be obtained as a mixture of Poisson and Gamma distributions. When the parameter p, the probability of success, is assumed to be unknown, the Beta and Pareto distributions are suitable for computing the compound distribution. In this paper, we propose a new model of compound binomial distribution by using the Inverse Gaussian distribution in order to model the heterogeneity of the population. This distribution has the advantage to be overdispersed and easy to implement in a variety of settings.

Palabras clave: Compound binomial distribution, Inverse Gaussian distribution, Overdispersed, Bonus–Malus premium.

Area temática: Métodos Cuantitativos.

1. Introduction

Negative Binomial distribution can be obtained as as a mixture of Poisson and Gamma distributions and this distribution has been traditionally used in many fields of social sciences. Modelling the number of claims in insurance markets or modelling consumption data are two common examples.

It is usually assumed that the probability p varies from individual to individual and has a prior distribution $\pi(\lambda)$ in the whole population. Traditionally the Beta and Pareto distribution, perhaps by mathematical convenience since they are conjugate to the likelihood function in the negative binomial distribution, have been used as prior distributions. When the parameter p, the probability of success, is assumed to be unknown, the Beta and Pareto distributions are suitable for computing the compound distribution (Alanko and Duffy (1996), Chatfield and Goodhardt (1970), Gómez and Vázquez (2003) and Shengwang et al. (1999)). The Pareto distribution presents several advantages. First of all it is conjugate to the negative binomial distribution, secondly it is unimodal and right skewed and finally it has mathematical flexibility for fitting different distribution pattern. As the parameter r, the number of success controls the extent of overdispersion of the individual claim distribution, approaches infinity the Negative Binomial–Pareto distribution approaches a Negative Binomial distribution (Meng and Withmore (1999) and Gómez and Vázquez (2003)). In this paper special attention is paid to the choice of the parameter p. We propose a new model of compound negative binomial distribution by using the Inverse Gaussian distribution. This distribution has thick tails, is unimodal and it also supplies the advantage of having closed form expression for the moment generating function (Tweedy (1957)). The parameters r and λ determines the extent of overdispersion. The larger r and λ are, the bigger the degree of overdispersion is.

The general model is presented in Section 2, where the marginal, the conditional expectation, the posterior distribution and some methods of estimation of parameters are showed. Section 3 is devoted to the Negative Binomial–Inverse Gaussian compound distribution as a particular example of the general model. An application as a particular insurance problem is presented in Section 4. Finally, conclusions are presented in Section 5.

2. Basic Results

A random variable Z has a inverse gaussian distribution if his pdf is given by,

$$f(z;\mu,\psi)=\left(\frac{\psi}{2\pi z^{3}}\right)^{1/2}\exp\left\{-\frac{\psi(z-\mu)^{2}}{2\psi^{2}z}\right\},\ z>0 \tag{1}$$

where $\psi,\mu>0$. We will represent $Z \sim IG(\mu,\psi)$. If $Z \sim IG(\mu,\psi)$, the moment generating function is given by,

$$M_{Z}(t)=E(e^{tZ})=\exp\left[\frac{\psi}{\mu}\left(1-\sqrt{1-2\mu^{2}t/\psi}\right)\right]. \tag{2}$$

Definition 1. We say that a random variable X has a negative binomial-inverse gaussian distribution if admits the stochastic representation:

$$X \mid \lambda \sim NB(r,p=e^{-\lambda}) \tag{3}$$

$$\lambda \sim IG(\mu,\psi), \tag{4}$$

with $r,\mu,\psi>0$. We will denote this distribution by $X \sim NBIG(r,\mu,\psi)$.

The next theorem stablishes the basic properties of this new distribution.

Theorem 1. Let $X \sim NBIG(r,\mu,\psi)$ be a negative binomial-inverse gaussian distribution defined in (3)-(4). Some basic properties are:

a) The probability mass function is given by,

$$\Pr(X=x)=\binom{r+x-1}{x}\sum_{j=0}^{x}(-1)^{j}\binom{x}{j}\exp\left\{\frac{\psi}{\mu}\left[1-\sqrt{1+\frac{2(r+j)\mu^{2}}{\psi}}\right]\right\} \tag{5}$$

with $x=0,1,2,\ldots$ and $r,\mu,\psi>0$.

b) The factorial moment of order k is given by

$$\mu_{[k]}(X) = E[X(X-1)\cdots(X-k+1)]$$

$$= \frac{\Gamma(r+k)}{\Gamma(r)} \sum_{j=0}^{k} (-1)^j \binom{k}{j} \exp\left\{ \frac{\psi}{\mu} \left[1 - \sqrt{1 - \frac{2(k-j)\mu^2}{\psi}} \right] \right\} \tag{6}$$

with $k = 1, 2, ...$

c) The mean and variance are,

$$E(X) = r\left[M_\lambda(1) - 1\right], \tag{7}$$

$$var(X) = (r + r^2)M_\lambda(2) - rM_\lambda(1) - r^2 M_\lambda^2(1), \tag{8}$$

where $M_\lambda(u)$ is defined in (2).

Proof. If $X \mid \lambda \sim NB(r, e^{-\lambda})$ and $\lambda \sim IG(\mu, \psi)$, the pdf of X can be obtained using the well-known compound formula,

$$\Pr(X = x) = \int_0^\infty \Pr(X = x \mid \lambda) f(\lambda; \mu, \psi) \mathrm{d}\lambda,$$

where $f(\lambda; \mu, \psi)$ is the pdf of a inverse gaussian distribution defined in (1). Now, the factorial moment of a negative binomial distribution is (see Balakrishnan and Nevzorov (2004)):

$$\mu_{[k]}(X \mid \lambda) = \frac{\Gamma(r+k)}{\Gamma(r)} \left(e^\lambda - 1\right)^k, \ k = 1, 2, ...$$

Finally, taking mathematical expectations and combining with (2) we obtain (6). □

The next theorem proofs the overdispersion of the negative binomial–inverse gaussian distribution and compares the moments of the new distribution with a negative binomial distribution with the same mean.

Theorem 2. Let $\lambda \sim IG(\mu,\psi)$ be a Inverse Gaussian distribution with pdf (1) and $\tilde{X} \sim NB(r, p=[E(e^{\lambda})]^{-1})$. Consider a negative binomial-inverse gaussian distribution X defined in (3)-(4). Then:

1. It is satisfies: $E(\tilde{X}) = E(X)$ and $var(X) > var(\tilde{X})$.
2. $var(X) > E(X)$.

Proof. Obviously $E(e^{\lambda}) > 1$, and then $p = 1/E(e^{\lambda})$ is well defined. Now, $E(X) = E[E(X \mid \lambda)] = r[E(e^{\lambda}) - 1]$, and from $var(X) = E[var(X \mid \lambda)] + var[E(X \mid \lambda)]$, we obtain that $var(X) = r[E(e^{2\lambda}) - E(e^{\lambda})] + r^2 var(e^l ambda)$. From $\tilde{X} \sim NB(r, p=[E(e^{\lambda})]^{-1})$, we have that $E(\tilde{X}) = r[E(e^{\lambda}) - 1]$ and $var(\tilde{X}) = r[E(e^{\lambda}) - 1]E(e^{\lambda})$. Now,

$$\begin{aligned} var(X) - var(\tilde{X}) &= E_{\lambda}[var(X \mid \lambda)] + var_{\lambda}[E(X \mid \lambda)] - var(\tilde{X}) \\ &= E[r(e^{\lambda} - 1)e^{\lambda}] + var[r(e^{\lambda} - 1)] - var(\tilde{X}) \\ &= r[E(e^{2\lambda}) - (E(e^{\lambda}))^2] + r^2 var(e^{\lambda}) \\ &= (r + r^2) var(e^{\lambda}) > 0 \end{aligned}$$

Finally, (1) is a direct consequence of (2). $\square$

Figure 1 shows some examples of probability mass functions of the NB–IG with different values of r, μ and ψ. This seems to be unimodal and skewness to the left in all cases considered.

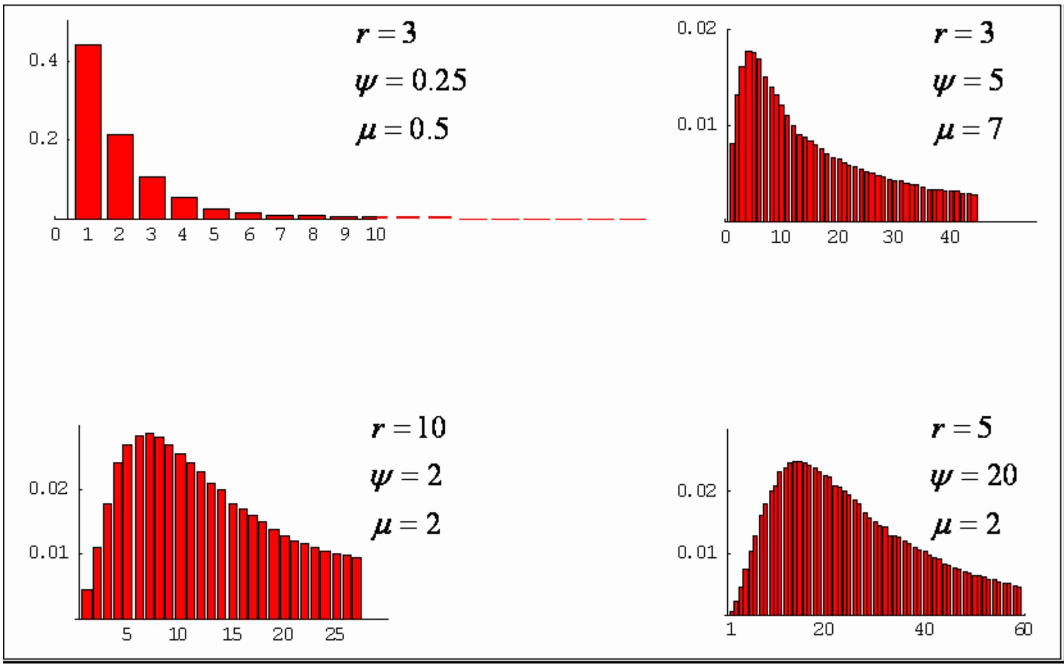

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Figure 1. Some examples of probability mass functions of the Negative Binomial–Inverse Gaussian random variable with different values of r, ψ and μ

3. Estimation of parameters

In this section different methods of estimation are given.

1. Moments method

Using expressions (7) and (8) together with the third moment about cero, which is given by

$$E[X^3] = (r^3 + 3r^2 + 2r)M_\lambda(3) - (3r + 6r^2 + 3r^3)M_\lambda(2) + (r + 3r^2 + 3r^3)M_\lambda(1) - r^3,$$

we can estimate the three parameters of the model.

Moment estimates may be computed by equating the sample and theoretical moments. In this case, we will need the first, second and third order moments of the compound distribution, which are given by the expressions

$$\mu_1 = r\left[e^{\frac{\psi}{\mu}\left(1-\sqrt{1-\frac{2\mu^2}{\psi}}\right)} - 1\right] \tag{18}$$

$$\mu_2 = (r+r^2)e^{\frac{\psi}{\mu}\left(1-\sqrt{1-\frac{4\mu^2}{\psi}}\right)} - (r+2r^2)\left(\frac{\mu_1}{r}+1\right) + r^2 \tag{19}$$

$$\mu_3 = (3r^3+3r^2+r)\left(\frac{\mu_1}{r}+1\right)$$

$$-(3r^3+6r^2+3r)\left[\frac{\mu_2+(1+2r)\left(\mu_1+r\right)-r^2}{r+r^2}\right]$$

$$+(r^3+3r^2+2r)e^{\frac{\psi}{\mu}\left[1-\sqrt{1-\frac{6\mu^2}{\psi}}\right]} - r^3 \tag{20}$$

By isolating the parameter ψ from equations (18) and (19) we obtain

$$\psi = \frac{\mu\log^2(\frac{\mu_1}{r}+1)}{2\log(\frac{\mu_1}{r}+1) - 2\mu} \tag{21}$$

and

$$\psi = \frac{\mu\log^2\left[\frac{\mu_2+(1+2r)(\mu_1+r)-r^2}{r+r^2}\right]}{2\log\left[\frac{\mu_2+(1+2r)(\mu_1+r)-r^2}{r+r^2}\right] - 4\mu} \tag{22}$$

respectively.

By equalling (21) and (23) and after computation, we get μ

$$\mu = \frac{\log\left[\frac{\mu_2+(1+2r)(\mu_1+r)-r^2}{r+r^2}\right]\log^2\left(\frac{\mu_1}{r}+1\right) - \log^2\left[\frac{\mu_2+(1+2r)(\mu_1+r)-r^2}{r+r^2}\right]\log(\frac{\mu_1}{r}+1)}{2\log^2(\frac{\mu_1}{r}+1) - \log^2\left[\frac{\mu_2+(1+2r)(\mu_1+r)-r^2}{r+r^2}\right]} \tag{23}$$

Substituting (21) or (22) and (23) in equation (20) we obtain an expression that only depends on the parameter r and can be solved numerically.

2. Maximum likelihood method

Estimation of parameters in the Negative Binomial compound model in (5) can be also accomplished via maximum likelihood. Denoting the parameters of the IG distribution by $\Theta = (\theta_1, \theta_2)$, the observed frequencies by $f_0, ..., f_n, f = \sum_{i=1}^{n} f_i$, and the corresponding probabilities from equation (5) by $p_0, ..., p_n$. We have that the log–likelihood function is given by

$$\log L(\phi) = \sum_{x=0}^{n} f_x \log p_x$$
$$= \sum_{x=0}^{n} f_x \log\left[\binom{r+x-1}{x} \sum_{j=0}^{x} \binom{x}{j} (-1)^{x-j} M_\lambda(-(r+j))\right], \quad \phi = (r, \Theta).$$

The maximum likelihood estimator $\hat{\phi}$ of ϕ is calculated using the following formulas:

$$\frac{\partial \log L(\phi)}{\partial \theta_i} = \sum_{x=0}^{n} \frac{f_x}{p_x} \frac{\partial}{\partial \theta_i}\left[\binom{r+x-1}{x} \sum_{j=0}^{x} \binom{x}{j} (-1)^j M_\lambda(-(r+j))\right]$$

$$= \sum_{x=0}^{n} \frac{f_x}{p_x} \sum_{j=0}^{x} \binom{x}{j} (-1)^j \binom{r+x-1}{x} \frac{\partial}{\partial \theta_i} M_\lambda(-(r+j)) = 0, i = 1, 2. \tag{24}$$

$$\frac{\partial \log(\phi)}{\partial r} = \sum_{x=0}^{n} \frac{f_x}{p_x} \sum_{j=0}^{x} \binom{x}{j} (-1)^j \frac{\partial}{\partial r}\left[\binom{r+x-1}{x} M_\lambda(-(r+j))\right] = 0. \tag{25}$$

Finally, in order to obtain the maximum likelihood estimates we have to solve the systems of equations given by (24) and (25). For that, we need the following derivatives:

$$\frac{\partial L(t)}{\partial \mu}=\left[-\frac{\psi}{\mu^2}\left(1-\sqrt{1+\frac{2\mu^2 t}{\psi}}\right)-2t/\sqrt{1+\frac{2\mu^2 t}{\psi}}\right]e^{\frac{\psi}{\mu}\left(1-\sqrt{1+\frac{2\mu^2 t}{\psi}}\right)}, \tag{26}$$

(27)

$$\frac{\partial L(t)}{\partial \psi}=\left[\frac{1}{\mu}\left(1-\sqrt{1+\frac{2\mu^2 t}{\psi}}\right)+\mu t/\left(\psi\sqrt{1+\frac{2\mu^2 t}{\psi}}\right)\right]e^{\frac{\psi}{\mu}\left(1-\sqrt{1+\frac{2\mu^2 t}{\psi}}\right)}, \tag{28}$$

(29)

$$\frac{\partial L(t)}{\partial r}=-\frac{\mu}{\sqrt{1+\frac{2\mu^2 t}{\psi}}}e^{\frac{\psi}{\mu}\left(1-\sqrt{1+\frac{2\mu^2 t}{\psi}}\right)}, \tag{30}$$

(31)

$$\frac{\partial}{\partial r}\left\{\frac{\Gamma(r+k)}{\Gamma(r)\Gamma(k+1)}\right\}=\frac{\Gamma(r)\Gamma^{'}(r+k)-\Gamma(r+k)\Gamma^{'}(r)}{\Gamma(k+1)\left[\Gamma(r)\right]^2}. \tag{32}$$

3. Zero proportion method

Because moment or likelihood methods can be too complicated in order to estimate the parameters of the model there is another way to estimate the parameters when the distribution has special features high. Zero proportion method tends to work well only when the mode of the distribution is at zero and its proportion of zeros is relatively high (Alanko and Duffy (1996)). The marginal or compound distribution depends on three parameters and therefore we will need three equations in order to estimate the parameters. These three equations are given by the proportion of zeros, p_0, $E\left(X\right)$ and $E\left(X^2\right)$. By equating sample and theoretical moments of first and second order we get two equations. Finally, the third equation can be obtained from (?) when $X=0$ and by equating sample proportion of zeros, $p_0=M_\lambda(-1)$.

Thus, by equalling the first, second order moments of the compound distribution and the sample moments and the theoretical and sample proportion of zeros we obtain the equations (18), (19) and

$$p_0 = P(K=0) = e^{\frac{\psi}{\mu}\left[1-\sqrt{1+\frac{2\mu^2 r}{\psi}}\right]}. \tag{33}$$

Again, by isolating the parameter ψ from equations (33) and (18) we obtain

$$\psi = \frac{\mu \log^2 p_0}{2\mu r + 2\log p_0} \tag{34}$$

and

$$\psi = \frac{\mu \log^2(\frac{\mu_1}{r}+1)}{2\log(\frac{\mu_1}{r}+1) - 2\mu}. \tag{35}$$

respectively.

By equating (34) and (35) and after computation, we obtain the parameter μ

$$\mu = \frac{\log^2 p_0 \log(\frac{\mu_1}{r}+1) - \log p_0 \log^2(\frac{\mu_1}{r}+1)}{r\log^2(\frac{\mu_1}{r}+1) + \log^2 p_0} \tag{36}$$

Again, substituting (34) or (35) and (36) in equation (19) we obtain an expression that only depends on the parameter r and can be solved numerically.

4. Applications

In order to show how the $NB-IG$ distribution works we have chosen two different examples. In the first example, data concerns the number of automobile liability policies in Switzerland for private cars (Klugman, et al. (1988), pp. 245). These data appears in Table 1 (first and second column).

Claims number	Observed	%	Fitted(1)	Fitted(2)	Fitted(3)	Fitted(4)
0	103704	86.52	103711	103708	103704	103704
1	14075	11.74	14051.1	14060.9	14071.5	14074.7
2	1766	1.47	1789.67	1779.25	1773.31	1770.8
3	255	0	251.32	254.2	253.26	251.95
4	45	0	40.17	41.08	41.45	41.76
5	6	0	7.31	7.32	7.61	7.98
6	2	0	1.50	1.40	1.53	1.72
More than 6	0	0	0	0	0	0
Total	119853	100	119853	119853	119853	119853
χ^2			0.5516	0.1655	0.0877	0.0825
d.f			2	2	2	2
p-value			0.7604	0.9210	0.9573	0.9598
Estimates			MM(1)	MM(2)	PM	MLE
$\hat{r}$			2.085	28.12	12.5	3.7381
$\hat{\psi}$			0.2253	0.0059	0.0122	0.075
$\hat{\mu}$			0.0709	0.0054	0.0148	0.0402

TABLE 1. Observed and fitted claims.

We can observe that moment estimation (MM), zero proportion method (PM) and maximum likelihood method (MLE) provide an almost perfect fit. In the MM we have found two solutions both providing a similar fit. Obviously, MLE gives us the best fit. Comparing this with the fits in Klugman et al. (1988), pp. 245, we can conclude that the $NB-IG$ distribution provides the best fit if we choose the χ^2–test as criteria of comparison. Finally, the log–likelihood is larger in the Binomial Negative–Inverse Gaussian distribution than in the Poisson and Negative Binomial and equal to the Poisson–Inverse Gaussian distribution.

Second data concerns again to the number of claims on automobile liability policies (Klugman et al. (1988), pp. 244). These data, which appear in Table 2 (first and second column) present more observations.

Number of accidents	Number of stretches	%	Fitted(1)	Fitted(2)
0	99	33.22	95.31	95.34
1	65	21.81	76.08	76.40
2	57	19.12	50.62	50.78
3	35	11.74	31.44	31.44
4	20	6.71	18.83	18.75
5	10	3.35	11.03	10.93
6	4	1.34	6.36	6.27
7	0	0.00	3.63	3.55
8	3	1.00	2.05	2.00
9	4	1.34	1.15	1.11
10	0	0.00	0.64	0.62
11	1	0.00	0.36	0.34
12	0	0.00	0.20	0.19
Total	298	100	298	298
χ^2			4.0085	3.99546
d.f			4	4
p-value			0.4143	0.4160
Estimates			MM	MLE
$\hat{r}$			1.5	1.5
$\hat{\psi}$			2224.82	3059.91
$\hat{\mu}$			0.760099	0.75091

TABLE 2. Observed and fitted accidents.

Similar conclusions to the first data are now obtained. In this case, zero proportion method was not used because this is only a 33.22% of the total observations and this method obviously works wrongly. Comparing with the Poisson, Negative binomial and Polya–Acepli distributions we can conclude that only this latest distribution works better than the $NB-IG$ distribution proposed.

1. An example computing automobile insurance premiums

In automobile insurance markets, Negative binomial is preferred to the Poisson since it is overdispersed and actual experience shows that this is certainly observed in the field of automobile insurance. When the number of claims, given the parameter $\lambda > 0$,is considered to be distributed according to $NB\left(r, r/(r+\lambda)\right)$ a natural prior for λ is the Generalized Pareto (GP) distribution, which is conjugated with respect to that likelihood (Gómez and Vázquez, 2003). Because the $NBIG$ distribution is overdispersed and in order to use other compound distribution than the $NB-GP$,we have chosen the $NBIG$ for computing automobile insurance premiums. Although the IG distribution is not conjugate with the likelihood NB we can see that the posterior distribution is easily obtained by dividing the mixing distribution by the marginal distribution as follows.

$$\begin{aligned}
f(\lambda \mid \mathbf{x}) &= \frac{\Pr(\mathbf{x} \mid \lambda) f(\lambda)}{\int_{\Lambda} \Pr(\mathbf{x} \mid \lambda) f(\lambda) d\lambda} \\
&= \frac{\sum_{j=0}^{\mathbf{x}} (-1)^j \binom{\mathbf{x}}{j} e^{-\lambda(j+rn)} \pi(\lambda)}{\sum_{j=0}^{\mathbf{x}} (-1)^j \binom{\mathbf{x}}{j} M(-(rn+j))} \\
&= \frac{\sum_{j=0}^{\mathbf{x}} (-1)^j \binom{\mathbf{x}}{j} \sqrt{\frac{\psi}{2\pi\lambda^3}} \exp\left\{-\lambda(j+rn) - \frac{\psi(\lambda-\mu)^2}{2\mu^2\lambda}\right\}}{\sum_{j=0}^{\mathbf{x}} (-1)^j \binom{\mathbf{x}}{j} \exp\left\{\frac{\psi}{\mu}\left[1 - \sqrt{1 + \frac{2(r+j)\mu^2}{\psi}}\right]\right\}}.
\end{aligned}$$

In Europe it is common to use bonus–malus in the automobile insurance premiums. In bonus–malus systems the premium depends only on the number of claims K caused by the policyholder in the past, irrespective of their size. The methodology of a bonus–malus system consists of ensuring that the premium increases with the number of claims and decreases with the period nt in which the policyholder does not make a claim. The premium for the first year is a priori premium because there is no information concerning the risk. Under quadratic loss function, i.e. using net premium (Gómez and Vázquez (2003)) the premium can be computed as $E_f\left(\delta(\lambda)\right)$. Here $\delta(\lambda)$ is the unknown risk premium because the parameter λ is unknown.

For the n th year we take into account the information about the number of claims during the first n years. Assuming the sequence of claims $x_1, x_2, ..., x_n$ over n years (independent and identically distributed) and letting $\mathbf{x} = \sum_{i=1}^{n} x_i = n\overline{x}$, the Bayes or experience ratemaking premium can be obtained as $\delta(\mathbf{x}, n) = E_{f(\lambda|\mathbf{x})}\left(\delta(\lambda)\right)$. If the first premium is 100, we can construct a bonus–malus table of premiums depending on $\mathbf{x}$ and n with the expression

$$\delta^*(\mathbf{x}, n) = 100\frac{E_{f(\lambda|\mathbf{x},n)}\left(\delta(\lambda)\right)}{E_{f(\lambda)}\left(\delta(\lambda)\right)} = 100\frac{\delta(\mathbf{x}, n)}{\delta(0,0)}.$$

k							
n	0	1	2	3	4	5	6
0	100						
1	92.52	138.56	199.44	271.62	350.97	434.47	520.38
2	86.47	126.82	179.62	242.12	310.96	383.59	458.47
3	81.50	117.34	163.94	218.95	279.65	343.83	410.13
4	77.26	109.59	151.19	200.26	254.46	311.90	371.32
5	73.65	103.03	140.61	184.85	233.76	285.68	339.47
6	70.46	97.43	131.67	171.91	216.43	263.75	312.87
100	23.55	26.60	30.00	33.73	37.78	42.11	46.69
1000	7.62	7.94	8.27	8.62	8.98	9.35	9.74
10000	2.41	2.45	2.48	2.51	2.55	2.58	2.61

TABLE 3 BMP under negative binomial-Inverse Gaussian model.

5. Conclusions and extensions

There are several other applications to which this new model presented above may be appropriate. On one hand, the compound distribution can be extended to a multivariate version. The Negative Binomial- Inverse Gaussian distribution admits a multivariate version which is a natural version of the univariate case. Furthermore, when stationary and conditional independence is assumed on the parameter λ the compound distribution can be computed and therefore, conditional predictions as well. On the other hand, we could have used the Generalized Inverse Gaussian as a mixing distribution. In this case four parameters should be estimated in the univariate case.

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